Iterative methods

# Fixed point theorem

## Exercise 1:

How many iterations need to calculate ?

### Iteration

starting value:

first iteration:

How many iterations need to calculate ?

We need at least 25 steps.

## Exercise 2:

Is fixed point iteration being convergence (can we calculate it)?

### Convergence

row norm below 1 => iteration will convergence

## Exercise 3:

Is fixed point iteration being convergence (can we calculate it)?

### Convergence

row norm above 1 => iteration will divergence

## Exercise 4:

Is fixed point iteration being convergence (can we calculate it)?

If convergence how many iterations are we need to ?

### Convergence

row norm below 1 => iteration will convergence

### Iteration

How many iterations need to calculate ?

starting value:

first iteration:

88 iteration needed.

## Exercise 5 (H.W.):

Is fixed point iteration being convergence (can we calculate it)?

If convergence how many iterations are we need to ?

### Convergence

row norm equal 1

row norm below 1 => iteration will convergence

### Iteration

How many iterations need to calculate ?

starting value:

first iteration:

need 76 iterations.

In reality 18 iterations are enough.

# Richardson iteration

## Exercise 1:

### Convergence

A is self-join:

A is positive definite:

A is convergence

### Calculate

I. Calc eigenvalues

II. Calc iteration

### Iteration count

How many iterations need to calculate ?

starting value:

first iteration:

## Exercise 2:

### Convergence

A is self-join:

A is positive definite:

A is convergence

### Calculate

I. Calc eigenvalues

II. Calc iteration

Matlab:

>> A=[4 -1 0; -1 4 -1; 0 -1 4]

>> w=2/8

>> I = [1 0 0; 0 1 0; 0 0 1]

>> x = [0 0 0]

>> x = x.'

>> x = (I-w.\*A)\* x + (w\*b).'

# Jacobian iteration

## Exercise 1:

Is A being convergent?

### Check diagonal dominant

A diagonally dominant

### Check transition (B) matrix

row

col

Matrix is convergernt

## Exercise 2:

### Check diagonal dominant

Not diagonal dominant

### Check transition (B) matrix

row

col

not below 1

### Check spectral radius

Eigenvalues:

less then 1

Jacobian iteration convergence

## Exercise 3:

### Check diagonal dominant

A is NOT diagonal dominant

### Check transition (B) matrix

Norm not less then 1

### Check spectral radius

Eigenvalues:

spectral radius is not less then 1

Jacobian iteration is divergence

## Exercise 4:

### Check diagonal dominant

A is NOT diagonal dominant

### Check transition (B) matrix

Norm not less then 1

### Check spectral radius

Eigenvalues:

spectral radius is less then 1

Jacobian iteration is convergence

# Seidel iteration

## Exercise 1:

Is matrix convergent to Seidel iteration?

A is not self-adjoint.

### Check diagonal dominant:

Diagonal dominant

### Calculate transition (B) matrix

### Check transition (B) matrix

Siedel is convergent.

## Exercise 2:

Is matrix convergent to Seidel iteration? We know that Jacobi iteration is divergence.

A is self-adjoint.

### Check diagonal dominant:

Not diagonal dominant

### Calculate transition (B) matrix

### Check transition (B) matrix

Cannot decide convergent.

### Check spectral radius

Seidel is convergent

## Exercise 3:

Is matrix convergent to Seidel iteration? We know that Jacobi iteration is divergence.

A is not self-adjoint.

### Check diagonal dominant:

Not diagonal dominant

### Calculate transition matrix

Seidel is divergent